STUDY OF SCATTERING & RESULTANT RADIATION PATTERN: INFINITE LINE CURRENT SOURCE POSITIONED HORIZONTALLY OVER A PERFECTLY CONDUCTING INFINITE GROUND PLANE

IMPLEMENTATION OF ANALYTICAL (MATLAB) AND NUMERICAL (HFSS) SOLUTIONS

ADVANCED ELECTROMAGNETIC THEORY

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1 Infinite Electrical Line Source above Infinite Plane Electric Conductor:

When a infinite electric line source is placed at a height ‘h’ above an infinite plane electrical conductor, as shown in figure 1-1, the solution for field components must include the presence of conducting plane. This can be accomplished by using Maxwell’s equations or the wave equation subject to the radiation conditions at infinity and the boundary condition along the air-conductor interface. Instead of doing this, the same solution can be more easily obtained by introducing an equivalent model to calculate these fields in the region of interest. Since the field below the interface (below infinite plane electric conductor, \( y < 0 \)) are known (they are zero), the equivalent model based on “image theory” leads to same fields as the actual physical problem, on or above the interface (\( y \geq 0 \)).

The equivalent problem of figure 1-1 (original geometry) is formulated using image theory in figure 1-2 where the ground plane has been replaced by an equivalent source (usually referred to as image or virtual source). The image source is introduced to account for the reflection from surface of the ground plane. The magnitude, phase, polarization and position of image source must be such that the boundary conditions of the equivalent problem of figure 1-2 along \(-\infty \leq x \leq +\infty\) are the same as those of physical problem of figure 1-1. In this situation the image must have: the same magnitude as the actual source, its phase must be 180˚ out of phase from actual source, it must be placed below the interface at depth \( h \) (\( y = -h \)) along a line perpendicular to the interface and passing though the actual source, and its length must also be parallel to the z-axis. Such a system configuration, as shown in figure 1-2, leads to zero tangential electric field along \(-\infty \leq x \leq +\infty\) which is identical to that of figure 1-1 along the air-electric conductor interface.

Therefore according to figure 1-1 or 1-2, the total electric field is equal to

\[
E^t = E^i + E^r = \begin{cases} \frac{-\hat{\alpha}_z}{4\omega \varepsilon} \left[ H_0^{(1)}(\beta \rho_i) - H_0^{(1)}(\beta \rho_r) \right] & y \geq 0 \\ 0 & y < 0 \end{cases} \quad (1a)
\]

Which for observations at large distance (far field), as shown in figure 1-3, reduces to:

\[
E^t = E^i + E^r = \begin{cases} -\hat{\alpha}_z \eta l e \left( \frac{e^{-j\beta \rho_i}}{\sqrt{\rho_i}} - \frac{e^{-j\beta \rho_r}}{\sqrt{\rho_r}} \right) \sqrt{\frac{j\beta}{8\pi}} & y \geq 0 \\ 0 & y < 0 \end{cases} \quad (2a)
\]
According to figure 1-3, for observation made at large distances ($\rho >> h$)

$$\rho_i \approx \rho - h \cos(\frac{\pi}{2} - \phi) = \rho - h \sin(\phi)$$
$$\rho_r \approx \rho + h \cos(\frac{\pi}{2} - \phi) = \rho + h \sin(\phi)$$

\textit{for phase variation} \hspace{8cm} (3a)

$$\rho_i \approx \rho_r \approx \rho$$

\textit{for amplitude variation} \hspace{8cm} (3b)

These approximations usually referred to in antenna and scattering theory as the far-field approximations. Using 3a and 3b, we can reduce 2a and 2b to

$$E^t = E^l + E^r = \begin{cases} 
-\bar{\alpha}_z \eta I_e \frac{j\beta}{8\pi} \big( e^{i\beta h \sin \phi} - e^{-i\beta h \sin \phi} \big) \frac{e^{-i\beta \rho}}{\sqrt{\rho}} \\
0
\end{cases} \begin{array}{c} y \geq 0 \hspace{7cm} (4a) \\
y < 0 \hspace{7cm} (4b)
\end{array}$$

Normalized amplitude E-field pattern (in decibels) for an infinite line source placed at a height of $h = 0.25\lambda$ and $0.5\lambda$ above the plane are shown in Figure 1-4.
Figure 1-3: Applied Image Theory (Far Field)

Figure 1-4: Radiation Pattern of an infinite Line source above an infinite Conducting plane
2 Modeling and Results in Matlab

The analytical results derived in the last section were implemented using equations 1a, 1b and 3a without using approximations (far field etc) in Matlab for the following two conditions:

1. Infinite Wire above Infinite Ground Plane at height of 0.25 \( \lambda \).
2. Infinite Wire above Infinite Ground Plane at height of 0.5 \( \lambda \).

These conditions were simulated in Matlab code for Free Space (Air)-Conductor Interface at following parameters:

Current = 1mA

Frequency = 3GHz

Distance of Observation Point = 4 \( \lambda \) (Far Field Transition)

Note: The 4 \( \lambda \) observation point was selected (as good enough approximation for far field) because the calculation range for HFSS beyond 4 \( \lambda \) requires excessive amount of computational resources and time. Hence to create a comparison between Matlab and HFSS results the simulation was conducted at 4 \( \lambda \) in both softwares.

The Matlab code is attached as an Annexure A.

Results are obtained in Matlab for Normalized E-Field (incident and total) and plotted in a polar plot (Figure 2-1 and 2-2) versus angle \( \phi \) for two heights 0.25 \( \lambda \) and 0.5 \( \lambda \). These plots agree well with those of Figure 1-4. The Matlab results are simply normalized instead of using decibel scale due to inherent limitation in Matlab on drawing (–inf) at 0° and 180° as well as plotting negative decibel values in a polar plot.

It may be noted that incident E field is off-centered with respect to polar plot/conducting plane, due to line source being located at height \( h \) above conducting plane. The lower half of incident E field i.e. from 180° to 360° is reflected back from conducting plane and adds with incident E field from 0° to 180° to result is total radiation pattern in each case.

It is emphasized that the choice of constant parameters (as above) in the Matlab code has no effect on Radiation pattern since all dimensions are varied in terms of wavelength and Radiation pattern is plotted as normalized as well.
Radiation Patterns of Wire for Different Values of Heights from Ground Plane (0.25λ & 0.5λ)

Figure 2-1: Total Radiation Patterns for height of 0.25λ and 0.5λ (Matlab Plot)
3 Modeling & Results in HFSS:

HFSS (High Frequency Structure Simulator) software is dedicated full-wave electromagnetic field software for electromagnetic problems employing FEM (Finite Element Method) solver. Geometry of the problem involving infinite line source of infinitesimal thickness and interface with an infinite conducting plane does not render it very suitable for numerical solution using HFSS (which employs Finite Element Method).

The wire model is placed along z-axis above the infinite ground plane as shown in figure 3-1. The length of the wire cannot be assumed infinite in HFSS hence we had to limit its size to 10 λ. The radius of the wire was taken as λ/100 but the results didn’t vary as long as the radius was less than λ/10. The wire was excited using a current source. As the current is a driven terminal source hence we can model this problem as Driven Terminal in the HFSS solution setup. The current source has to be assigned a path direction along the cylindrical wire we have modeled.

The wire is placed on a perfectly conducting plane of infinite size. That was modeled using a rectangular block with length and width of 20 λ and thickness of λ/200 (to model a thin perfectly conducting surface). This conducting plane was further made infinite by selecting the option of Infinite Ground Plane by assigning this plane as a Perfect E Boundary as shown in figures 3-2 to 3-4.

The third important model was the radiation boundary of the problem. For that purpose a cube of large dimensions were used but due to large mesh and computational restraints the boundary was assigned a height of 10 λ along z-axis and widths of 4 λ along x and y axis as shown in fig 3-5.

The EM problem was modeled in HFSS using the following parameters:

1. Frequency = 3 GHz
2. Wavelength = λ (0.1m)
3. Length of cylindrical wire = 10 λ (As the wire could not be modeled as an infinite wire in HFSS)
4. Radiation Boundary with Height of 10 λ and width of 4 λ (Radiation boundaries larger than this required large amount of computational resources and time. Causes Errors such as [warning] Solution Setup ‘Setup1’: Given the specified frequency and model dimensions, an extremely large mesh will be required to produce an accurate solution. Your model and/or frequency units may be set incorrectly.)
5. The current source for the cylindrical wire is used as 1mA and its position starts at center of the lower face and ends at center of the upper face.
6. The solution mesh is improved by λ/3 by default (by the software) in every iteration cycle to improve accuracy.
7. The end results were plotted as normalized in HFSS as shown in figures 3-6 and 3-7.

Normalized results obtained using above parameters through a numerical solver (HFSS) display great resemblance to analytical solution being plotted via Matlab code as well as curves in Figure 1-4 (obtained by Balanis [1])
Figure 3-1: Wire Length of 10\(\lambda\), Wire Radius \(\lambda/100\)

Figure 3-2: Side View of Wire Over Infinite Plane

Figure 3-3: Front View of Wire over Infinite Plane
Figure 3-4: Top View of Wire over Infinite Plane

Figure 3-5: Radiation Boundary over Infinite Ground Plane with wire along z-axis
Radiation Patterns of Wire for Different Values of Heights from Ground Plane

(0.25λ & 0.5λ)

Figure 3-6: For Line Source Height of 0.25λ over Plane (HFSS)

Figure 3-7: For Line Source Height of 0.5λ over Plane (HFSS)
4  Annexure A:

% Scattering of EM field of Infinite_Line_Source_over_Conducting_Plane
% Run Script M-File as EMscatter.m
% Plots scattered EM field for case of horizontally situated infinite
% Electric line source over an infinite Electric Conducting Plane
% for heights of 0.25*lambda and 0.5*lambda
% Reference: Advanced Engineering Electromagnetics by Constantine A.
% Balanis, First Edition 1989, Chapter 11, Section 11.2.3, Equations
% 11.10a, 11.17a,b and 11.19a & Figure 11-3
% Script M-File Created by Hammad Tanveer, Sohaib Afridi and Zunnurain
% on 8 Jan 2011 for Dr Ihsan Elahi's Assignment on Scattering for EMT
% course

% Constant Parameters are assumed, which however have no effect on
% Radiation pattern since all dimensions are varied in terms of wavelength
% and Radiation pattern is plotted as normalized.

clc
Ie=1e-3;  % Current in the line source assumed as 1mA
c=3e8;   % Speed of EM waves in Free Space
f=3e9;   % Frequency of EM oscillations assumed as 3GHz
lambda=c/f;
B=2*pi/lambda;
w=2*pi*f;
eps=8.854e-12;  % Electric Permittivity of Free Space
rho=4*lambda;  % Observation Point at 4*lambda (Far Field Transition)

i=0;
for h=0.25*lambda:0.25*lambda:0.5*lambda;
  i=i+1;
  phi=0:0.01*pi:pi;
  rhoi=rho-h*cos(0.5*pi-phi);
  rhor=rho+h*cos(0.5*pi-phi);
  Ei=(-B^2*Ie/(4*w*eps)).*besselh(0,2,B*rhoi);
  Er=(B^2*Ie/(4*w*eps)).*besselh(0,2,B*rhor);
  Et1=Ei+Er;
  EiNorm=abs(Ei)/max(abs(Et1));  % Normalized Incident E field
  Et1Norm=abs(Et1)./max(abs(Et1));  % Normalized total E field

  figure(1);
  subplot(1,2,i); polar(phi,Et1Norm);
  title([{'Height = ',num2str(i*0.25),'
  
  lambda'},'FontWeight', 'Bold']);
  end;

figure(2);
polar(phi,EiNorm);
title([{'Incident Field'},'FontWeight', 'Bold']);